

A Level Further Mathematics B (MEI)

Y420 Core Pure

Sample Question Paper

Version 3.0

Date – Morning/Afternoon

Time allowed: 2 hours 40 minutes

You must have:

- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You may use:

- a scientific or graphical calculator



INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION

- The total number of marks for this paper is **144**.
- The marks for each question are shown in brackets [].
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **24** pages. The Question Paper consists of **8** pages.

Section A (33 marks)

Answer **all** the questions.

1 Find the acute angle between the lines with vector equations $\mathbf{r} = \begin{pmatrix} 3 \\ 0 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$. [3]

2 (i) On an Argand diagram draw the locus of points which satisfy $\arg(z - 4i) = \frac{\pi}{4}$. [2]

(ii) Give, in complex form, the equation of the circle which has centre at $6 + 4i$ and touches the locus in part (i). [4]

3 Transformation M is represented by matrix $\mathbf{M} = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$.

(i) On the diagram in the Printed Answer Booklet draw the image of the unit square under M . [2]

(ii) (A) Show that there is a constant k such that $\mathbf{M} \begin{pmatrix} x \\ kx \end{pmatrix} = 5 \begin{pmatrix} x \\ kx \end{pmatrix}$ for all x . [2]

(B) Hence find the equation of an invariant line under M . [1]

(C) Draw the invariant line from part (ii) (B) on your diagram for part (i). [1]

4 You are given that $z = 1 + 2i$ is a root of the equation $z^3 - 5z^2 + qz - 15 = 0$, where $q \in \mathbb{R}$.

Find

- the other roots,
- the value of q . [5]

5 (i) Express $\frac{2}{(r+1)(r+3)}$ in partial fractions. [2]

(ii) Hence find $\sum_{r=1}^n \frac{1}{(r+1)(r+3)}$, expressing your answer as a single fraction. [5]

- 6 (i) A curve is in the first quadrant. It has parametric equations $x = \cosh t + \sinh t$, $y = \cosh t - \sinh t$ where $t \in \mathbb{R}$. Show that the cartesian equation of the curve is $xy = 1$. [2]

Fig. 6 shows the curve from part (i). P is a point on the curve. O is the origin. Point A lies on the x -axis, point B lies on the y -axis and OAPB is a rectangle.

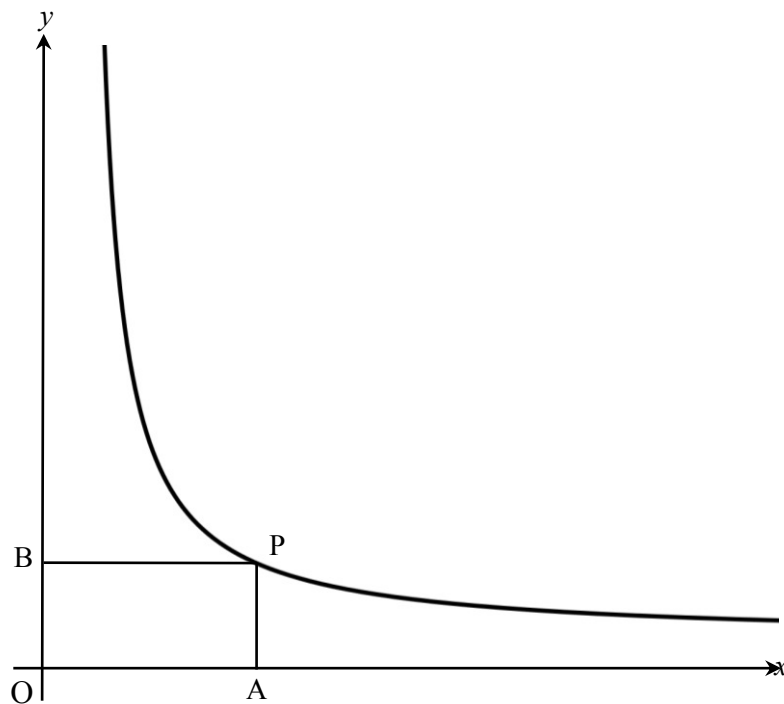


Fig. 6

- (ii) Find the smallest possible value of the perimeter of rectangle OAPB. Justify your answer. [4]

Section B (111 marks)

Answer **all** the questions

- 7 (i) Use the Maclaurin series for $\ln(1+x)$ up to the term in x^3 to obtain an approximation to $\ln 1.5$. [2]
- (ii) (A) Find the error in the approximation in part (i). [1]
- (B) Explain why the Maclaurin series in part (i), with $x=2$, should not be used to find an approximation to $\ln 3$. [1]
- (iii) Find a cubic approximation to $\ln\left(\frac{1+x}{1-x}\right)$. [2]
- (iv) (A) Use the approximation in part (iii) to find approximations to
- $\ln 1.5$ and
 - $\ln 3$. [3]
- (B) Comment on your answers to part (iv) (A). [2]
- 8 Find the cartesian equation of the plane which contains the three points $(1, 0, -1)$, $(2, 2, 1)$ and $(1, 1, 2)$. [5]
- 9 A curve has polar equation $r = a \sin 3\theta$ for $-\frac{1}{3}\pi \leq \theta \leq \frac{1}{3}\pi$, where a is a positive constant.
- (i) Sketch the curve. [2]
- (ii) **In this question you must show detailed reasoning.**
- Find, in terms of a and π , the area enclosed by one of the loops of the curve. [5]
- 10 (i) Obtain the solution to the differential equation
- $$x \frac{dy}{dx} + 3y = \frac{1}{x}, \text{ where } x > 0,$$
- given that $y=1$ when $x=1$. [7]
- (ii) Deduce that y decreases as x increases. [2]

- 11 (i) It is conjectured that

$$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \dots + \frac{n-1}{n!} = a - \frac{b}{n!},$$

where a and b are constants, and n is an integer such that $n \geq 2$.

By considering particular cases, show that if the conjecture is correct then $a = b = 1$.

[2]

- (ii) Use induction to prove that

$$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \dots + \frac{n-1}{n!} = 1 - \frac{1}{n!} \text{ for } n \geq 2.$$

[7]

- 12 In this question you must show detailed reasoning.

- (i) Given that $y = \arctan x$, show that $\frac{dy}{dx} = \frac{1}{1+x^2}$.

[3]

Fig. 12 shows the curve $y = \frac{1}{1+x^2}$.

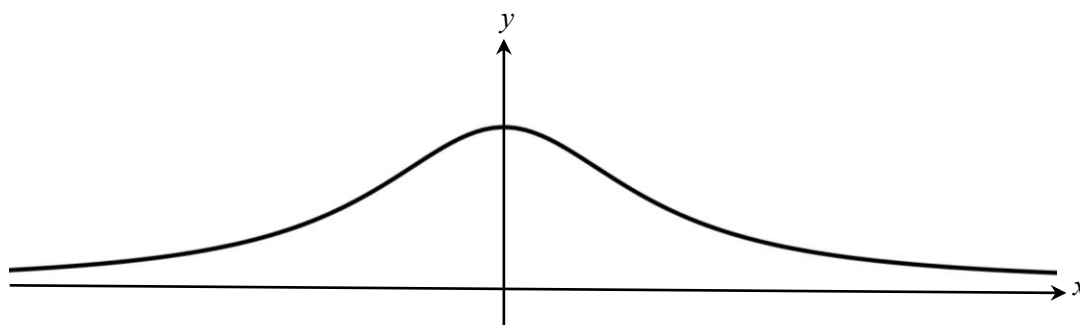


Fig. 12

- (ii) Find, in exact form, the mean value of the function $f(x) = \frac{1}{1+x^2}$ for $-1 \leq x \leq 1$.

[3]

- (iii) The region bounded by the curve, the x -axis, and the lines $x = 1$ and $x = -1$ is rotated through 2π radians about the x -axis. Find, in exact form, the volume of the solid of revolution generated.

[7]

13 Matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} k & 1 & -5 \\ 2 & 3 & -3 \\ -1 & 2 & 2 \end{pmatrix}$, where k is a constant.

(i) Show that $\det \mathbf{M} = 12(k - 3)$.

[2]

(ii) Find a solution of the following simultaneous equations for which $x \neq z$.

$$4x^2 + y^2 - 5z^2 = 6$$

$$2x^2 + 3y^2 - 3z^2 = 6$$

$$-x^2 + 2y^2 + 2z^2 = -6$$

[3]

(iii) (A) Verify that the point $(2, 0, 1)$ lies on each of the following three planes.

$$3x + y - 5z = 1$$

$$2x + 3y - 3z = 1$$

$$-x + 2y + 2z = 0$$

[1]

(B) Describe how the three planes in part **(iii) (A)** are arranged in 3-D space. Give reasons for your answer.

[4]

(iv) Find the values of k for which the transformation represented by $\mathbf{M}$ has a volume scale factor of 6.

[3]

14 (i) Starting with the result

$$e^{i\theta} = \cos \theta + i \sin \theta,$$

show that

$$(A) \quad (\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta \quad [2]$$

$$(B) \quad \cos \theta = \frac{1}{2}(e^{i\theta} + e^{-i\theta}). \quad [2]$$

(ii) Using the result in part **(i)** (A), obtain the values of the constants a , b , c and d in the identity

$$\cos 6\theta \equiv a \cos^6 \theta + b \cos^4 \theta + c \cos^2 \theta + d. \quad [6]$$

(iii) Using the result in part **(i)** (B), obtain the values of the constants P , Q , R and S in the identity

$$\cos^6 \theta \equiv P \cos 6\theta + Q \cos 4\theta + R \cos 2\theta + S. \quad [5]$$

(iv) Show that $\cos \frac{\pi}{12} = \left(\frac{26 + 15\sqrt{3}}{64} \right)^{\frac{1}{6}}.$ [3]

15 In this question you must show detailed reasoning.

Show that

$$\int_0^{\frac{2}{3}} \operatorname{arsinh} 2x \, dx = \frac{2}{3} \ln 3 - \frac{1}{3}. \quad [8]$$

- 16** A small object is attached to a spring and performs oscillations in a vertical line. The displacement of the object at time t seconds is denoted by x cm.

Preliminary observations suggest that the object performs simple harmonic motion (SHM) with a period of 2 seconds about the point at which $x = 0$.

- (i) (A)** Write down a differential equation to model this motion. **[3]**

- (B)** Give the general solution of the differential equation in part **(i) (A)**. **[1]**

Subsequent observations indicate that the object's motion would be better modelled by the differential equation

$$\frac{d^2x}{dt^2} + 2k \frac{dx}{dt} + (k^2 + 9)x = 0 \quad (*)$$

where k is a positive constant.

- (ii) (A)** Obtain the general solution of (*). **[3]**

- (B)** State two ways in which the motion given by this model differs from that in part **(i)**. **[2]**

The amplitude of the object's motion is observed to reduce with a scale factor of 0.98 from one oscillation to the next.

- (iii)** Find the value of k . **[3]**

At the start of the object's motion, $x = 0$ and the velocity is 12 cm s^{-1} in the positive x direction.

- (iv)** Find an equation for x as a function of t . **[4]**

- (v)** Without doing any further calculations, explain why, according to this model, the greatest distance of the object from its starting point in the subsequent motion will be slightly less than 4 cm. **[2]**

END OF QUESTION PAPER

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